About the structure of the Froissart limit in QCD

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Abstract

The Froissart asymptotic behavior of cross-sections is usually considered in a parton picture as corresponding to the collision of two almost black disks filled with partons. In this article we mainly concentrate on the examination of the local transparency of such F-disks. We discuss how is it possible to guarantee the boost-invariance of the reciprocal transparency of two such F-disks in a process of their collision, despite the fact that the mean area of the overlapping of these F-disks at the same impact parameter is varying with the Lorentz frame. We argue that on will always have such problems, if the dominant interactions at all energies remain soft, but such a trouble can be probably avoided if the mean parton virtualities grow with energies. This last is natural in QCD, and we use the qualitative generalization of BFKL approach to estimate the distribution of hard partons with various virtualities inside a F-disk. As a result, the quasiclassical partonic wave function corresponding to the F-limit can be approximately represented by the system of enclosed parton-gluon disks with a growing virtuality and blackness. With the increase of energy the new disks with larger virtualities are created in the middle of the previous disks, and then they expand with the same transverse velocity.

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1. Introduction.

This section has mainly a review character. We draw here attention to some problems connected with the dynamical structure of the "Froissart disk" and their description in terms of different approaches.

The Froissart ($\mathcal{F}$) [1] asymptotic behavior [1] corresponds to the growth of total cross sections like $\sigma_{tot} \sim Y^2$, where $Y = \log\ s$. This behavior is proved to be maximally fast in field theories with massive particles. In Regge-Glauber approach the $\mathcal{F}$ behavior arises naturally after the s-chanal unitarisation in models with the supercritical pomeron ($\mathcal{P}$) - with the trajectory $\alpha(t) \simeq 1+\Delta+\alpha' t, \quad \Delta > 0$. This follows already for a simple eiconal set of diagrams [2]. In QCD the perturbative $\mathcal{P}$ corresponds to the BFKL-like generalization [3] of a two gluon exchange, and is very likely supercritical. All exsisting experimental data on high energy hadron cross sections also point on a supercritical $\mathcal{P}$. Due to all this, it is accepted to believe that the $\mathcal{F}$ behavior of cross-sections becomes apparent when energies become essentially large.

The supercritical $\mathcal{P}$, when we write its contribution to an elastic scattering amplitude in the representation of impact parameters as

$$v(Y, x_\perp) = \frac{g_a g_b}{\alpha' Y_1} \cdot \exp(\ \Delta Y_1\ -\ x_\perp^2/4\pi\alpha' Y_1\ )\ , \tag{1.1}$$

where $Y_1 = Y + i\pi/2$, corresponds, after an eiconal-like unitarisation of the S-matrix, to the expressions

$$\begin{aligned} S(Y, x_\perp)\ &=\ e^{-v}, \qquad A(Y, x_\perp)\ =\ i(1 - e^{-v})\quad , \\ \sigma_{in}(Y, x_\perp)\ &=\ 1 - e^{-2\Re v}\ , \qquad \sigma_{tot}(Y, x_\perp)\ =\ 2(1 - \Re e^{-v})\ , \end{aligned} \tag{1.2}$$

which have a very distinctive property. For a large Y from (1.2) it follows approximately that

$$\sigma_{in}(Y, x_\perp)\ =\ (1 - T)\ \theta\Big(\ R(Y) - x_\perp\ \Big)\ , \tag{1.3}$$

where $R(Y) = Y \cdot 2\sqrt{\Delta\alpha'}$, and the transparency $T(Y, x_\perp) \to 0$ when $Y \to \infty$ and $x_\perp/R(Y) < 1$. This corresponds to an almost black disk with a radius $R(Y)$ whose border is spread by $\delta\ R(Y) \sim \sqrt{\Delta\alpha'}$. It expands linearly with Y, and thus leads to the $\mathcal{F}$ type behavior of cross-sections $\sigma_{tot} = 2\ \sigma_{in} \simeq \pi\ R^2(Y) \sim Y^2$.
The explicit eiconal dependence $S[v] = e^{-v}$ of S on v is by itself not essential to reach such conclusions, and instead of (1.2) one can in the same way work with generalized eiconal series

$$A(Y, x_\perp)\ =\ i \sum_{n=1}^{\infty} \frac{c_n}{n!} (-v)^n\ , \tag{1.4}$$

where c_n are nearly arbitrary positive coefficients, representing the contribution of diffraction generation beams. The general method of working with series (1.4), in a connection with the $\mathcal{F}$ behavior, was suggested in the nice work of Cardy [4], and by means of that one can in a simple way take into account also more complicated reggeon diagrams, not only a nonenhanced one (eiconal type). Various aspects of this approach are developed further in a number of papers [5, 6, 7, 8, 9]. The main point here is that we start from a black disk (of type (1.2) or (1.4)) as a zero approximation, and then find correction to it - this changes only slightly the whole picture. Corrections come from processes that take place on the borders of this $\mathcal{F}$ disk ($\mathcal{F}d$) - at $x_\perp \simeq R(Y)$ - when the impact parameters are such that two colliding $\mathcal{F}d$ touch each other by their spread borders. It corresponds to various diffraction generation and multipomeron processes, and one must take care that their cross-sections be not larger than the cross-sections of the main processes. Corrections to the internal parts of $\mathcal{F}d$ cancel in this approach due to an elastic screening [4], that is in fact the screening in the process of interaction.
The resulting picture of $\mathcal{F}$ corresponds to the soft and black $\mathcal{F}d$, and when interpreted in parton terms, is probably contradictory, as we discuss in this article.

In this paper we consider the transparency of the internal parts of $\mathcal{F}d$. This small quantity can be more sensitive to t-unitarity constraints, and is directly connected with the $S(Y, x_\perp)$-matrix, that gives the amplitude for a target to tunnel through $\mathcal{F}d$ at a given $x_\perp$ without an interaction - the corresponding transparency $T\ =\ |S|^2$.

[1] In this article we also use the sign $\mathcal{F}$ to denote the corresponding object(quasiparticle ?), which is sometimes called Froissaron

Let's begin with comments of the parton interpretation of some basic regge expressions corresponding to the supercritical $\mathcal{P}$. In the parton language the $\mathcal{P}$ contribution to $\Re v(Y, x_\perp)$ is proportional to the mean transverse density of low energy partons in a fast hadron. If the interactions of these partons with a small target are independent from each other, then the mean number of such interactions - $< n >$ is also proportional to $v(Y, x_\perp)$. Because in this (uncorrelated) case the probability of some number of interactions is given by Poison distribution, the probability of *no* interaction T is simply $\exp(- < n >)$, and this is what corresponds to the eiconal $S \sim \exp(-v)$, or to the close to (1.4) forms of the $S(Y,\ x_\perp)$-matrix.

The possible opposite situation is when the correlations, coming from large fluctuations in the target and in the incoming $\mathcal{F}d$, are maximale. For example, if the size r_t of the target fluctuates to small values, its interaction cross-section $\sigma_0(r_t)$ with all $\mathcal{F}d$ partons becomes small. As a result the mean number of interactions with the target in these configurations $< n > \sim v\sigma_0(r_t)$ can be also small. But the probability to come in this state can be larger than the probability to "not interact" e^{-2v} in the main configuration. In all QCD-like field-theories such probability for the target to fluctuate to small r_t, (in such a way to have $< n > \sim 1$), is of the order $\sim 1/v^2$. It then leads to

$$S \ = \ \frac{1}{1+v} \ , \qquad\qquad A \ = \ iv/(1+v) \tag{1.5}$$

Or one can consider such a fluctuation (in fact the corresponding small component of the stationary Fock function) in a fast hadron, that doesn't contain the $\mathcal{F}d$ at all. This probability is always $\sim \exp(-cY)$, that leads again to the power-like $S[v]$ of type (1.5). In terms of series (1.4) such a behavior corresponds to a fast grow of $c_n \ \sim \ n!$, and so leads to the S matrix decreasing much slowly with v [2]. But we still have a black disk at large v.

This example in particular shows that an elastic S matrix, although purely diffractive (the amplitude $A \ \simeq$ imaginary), can be defined at some $x_\perp$ not with main inelastic processes. And it is probably a rule and not an exclusion.

The essential point is that $v \to \infty$, $S[v] \to 0$ when $Y \to \infty$ - and all this needs the infinite growth of the parton density in a bare supercritical $\mathcal{P}$ with Y - as a result follows the blackness of the $\mathcal{F}d$. The various forms of $S[v]$ in (1.4) only represent the different patterns of screening of these partons one by another in process of their interaction with the target, when their density becomes large, and then the nearly black disk is used as a next approximation.

But let take firstly into account that partons interact - shadow one another and recombine in all rapidity interval and not only when they interact with the target. In the quasiclassical approximation (without $\mathcal{P}$ loops), to estimate this effect one must sum all tree diagrams with $\mathcal{P}$. After that one can (possibly) put the result in eiconal-like series (1.4), to take into account also the screening in the process of interaction with a target. For a large $\mathcal{F}d$ one can neglect the transverse motion (in $x_\perp$) of $\mathcal{P}$ inside the tree diagrams. Then the full contribution from the sum of all such diagrams can be represented in the simple form

$$v \to V(Y, x_\perp) \ \simeq \ \frac{v}{1+\lambda\ v} \ , \qquad \lambda \ \sim \ \frac{r_3}{\Delta} \ , \tag{1.6}$$

where r_3 is the $3\mathcal{P}$ vertex. In the one-dimensional parton language the Eq.(1.6) corresponds to the solution of the equation $\partial V/\partial y \ = \ \Delta V - r_3 V^2$ for the mean parton number V, where Δ gives the probability for a parton to split on an unit interval of rapidity, and r_3 the probability for two partons to recombine.

So, in this case, the amplitude $V(Y, x_\perp)$ and not v is proportional to the transverse parton density, and we have saturation, corresponding to the gray disk $V \to \lambda^{-1}$ at $Y \to \infty$, and not an infinite growth of the density. Then, instead of (1.1),(1.2), the "additionaly" unitarised S-matrix and the amplitude representing the gray $\mathcal{F}d$ take the form

$$S_1(b, y) = \sum_{n=0}^{\infty} \frac{c_n}{n!}(-V)^n \quad , \qquad F_1(b, y) \ = \ i\Big(1 \ - \ S_1(b, y)\Big) \ . \tag{1.7}$$

Here we can face with some problems, because higher order diagrams with $\mathcal{F}$ loops (containing V or F_1 as propagators, like in (1.7)), can give arbitrary larger powers of Y in amplitudes. And then one must again consider the full series of such diagrams and hope that it will converge to a sensible gray $\mathcal{F}$ limit (or now a black one ?), or consider all this as an evidence that the gray disk picture is inconsistent.

[2] Evidently for a large v such series don't converge and we must use some indirect method of summation of diagram contribution. This also shows that the decomposition over reggeon diagrams is not always the best way, alredy in such a simple situation.

But such arguments are slightly naive. Firstly because the series of diagrams with $\mathcal{F}$ as a quasiparticle are not the good ones, despite the fact that the series of all reggeon diagrams with $\mathcal{P}$ can be formally represented [4]as new series over the $\mathcal{F}$ like objects of type (1.4). It is because the initial(perturbatie) vacuum of a supercritical $\mathcal{P}$ is an unstable one, and the "quasiparticle" $\mathcal{F}$ represents in fact the growth with "time" y of a bubble of the other (stable) phase of the $\mathcal{P}$ system. If we write $\mathcal{P}$ Lagrangian in the form

$$\mathcal{L} \;=\; \psi^{+}\frac{\overleftrightarrow{\partial}}{2\,\partial y}\psi \;+\; \Delta\psi^{+}\psi \;+\; \alpha'\;\vec{\partial_{\perp}}\psi^{+}\;\vec{\partial_{\perp}}\psi \;+\; r_3\;(\psi^{+}\psi^{+}\psi+\psi^{+}\psi\psi)+\;...\;+(\psi^{+}J+J^{+}\psi)\;, \qquad (1.8)$$

where $\psi(y,x_{\perp})$ is $\mathcal{P}$ field and J - sources, representing "external" hadrons, then it is evident that at $\Delta>0$ the vacuum $\psi=0$ is unstable and the classically stable vacuum can be at $\psi=\Delta/r_3$. For the nonrelativistic system, described by (1.8), the initial vacuum is in $\psi=0$ state, and can not by itself tunnel to the other phase. But an external particle J can inject $\mathcal{P}$'s in the vacuum and then it leads to the growing bubble of other stable phase. As a result, in the diagrams with $\mathcal{F}$ loops the different $\mathcal{F}$'s (in fact "parallel condensats of ψ ") partially occupy the same place in $x_{\perp}$ plane during the "time" $y\sim Y$, with the area of $y*x_{\perp}$ overlapping $\mathcal{S}\sim Y^3$. Then one must probably add explicitly (or effectively - as a result of summation of some series of $\mathcal{F}$ subdiagrams) to every such a diagram a factor $\exp(-\mathcal{S})$ representing the probability that the "parallel" $\mathcal{F}$'s don't interact (like the Sudakov factor). And in this case the contributions of all diagrams with $\mathcal{F}$ loops would not lead to higher powers of Y, and can be essentiall only on the border of $\mathcal{F}d$ when $\mathcal{S}$ is small. This shows that the consistency of a gray picture of $\mathcal{F}d$ must be probably checked with another methods.
It should be noted, that the gray $\mathcal{F}d$ can be acceptable as an approximation (and possibly in a very broad energy interval) [10], before the hard component of $\mathcal{F}d$ becomes large.

It is known scene then, the strong coupling regime with the asymptotic behavior of cross sections of type $\sigma_{in}\sim Y^a$, $0\leq a\leq 2$ has been introduced [11], that in the limiting cases $a=0$, $a=2$ the infinite number of identities between vertices must be fulfilled, that is very unnatural, because it needs fine tuning of the infinite number of constants (bare vertices), coming from large distances. Despite that, let briefly mention the peculiar properties of such a $\mathcal{F}$. If we choose the $\mathcal{F}$ Green function and the $3\mathcal{F}$ vertices in the scaling form

$$\begin{aligned} &G_{\omega}(k) \;=\; \omega^{-3}\phi(k/\omega) \;\;, \\ &\Gamma_3(\omega,\omega_1,\omega_2,k_i)=\omega^3\gamma(k_i/\omega)\;, \end{aligned} \qquad (1.9)$$

(where $\omega=j-1$) then the Dyson equation for G and Γ_3 can be fulfilled on the level of their singular parts. The same is true for higher $\mathcal{F}$ vertices Γ_n. For example, the $3\Gamma_3$ term in the Dyson equation for Γ_3 reduces like

$$\int d\omega\; d^2k\; G^3\;\Gamma_3^3\sim\omega^3\sim\Gamma_3 \qquad (1.10)$$

All this also corresponds to the conditions, that can be described in a simbolical form as

$$G\Gamma_n \;=\; 1 \quad , \qquad (1.11)$$

and that leads to a cancelation of all "superfluous" poles in enhanced $\mathcal{F}$ diagrams, and therefore they are effectively reduced to unenhanced one. As a result the total cross section is given by the sum of multi-$\mathcal{F}$ exchange diagrams with all terms of the same order in Y, and one can hope that the rate of convergence of these series doesn't depend asymptotically on Y itself.
Because the corresponding $\mathcal{F}d$ is gray, the $3\mathcal{F}$ diffraction (of states with high masses) should be expected high ($\sim\sigma_{tot}$). And now it comes not only from the border of $\mathcal{F}d$. Using the Eq.(1.9) one can estimate that

$$\partial\sigma_{dif}/\partial\eta\sim\eta, \qquad (1.12)$$

where $\eta=\ln M_{beam}$. So one can expect that for such a $\mathcal{F}$ there are large fluctuations in individual events at the same impact parameter. Thus such a gray $\mathcal{F}d$ corresponds to a picture of a growing disk (bubble) filled by pomerons (or, in the other language, by corresponding partons), that are itself in a critical point state and not in a new stable phase like $\psi\sim\Delta/r_3$.

If we want to make transition from $\mathcal{P}$ to $\mathcal{F}$ using reggeon diagrams the main trouble is the following one. Reggeon diagrams can be safely applied only when the mean transverse density of reggeons is not a large

one - pomerons should not cover one another, because they are purely composite objects and can completely dissolve, when their density exceeds some critical value. This condition leads [12] to the "limiting" order $n_{max} \sim Y$ in all series over the reggeon diagrams, like (1.4). But in all expressions (1.2),(1.4) the mean essential $n \sim \exp Y\Delta$ is much higher then such n_{max} at $Y \gg 1/\Delta$, and so the prepared $\mathcal{F}$ takes in fact the main contribution from the values of n located outside the region of the $\mathcal{P}$'s applicability. And, as a result, if we cut series on the order $n \sim Y$, different methods of summation can give different answers.
The situation may be slightly better when the effective transverse size of $\mathcal{P}$'s decreases, when their density becomes so large that $\mathcal{P}$'s begin to touch one another, so that as a result they don't dissolve. Or, in other words, more and more hard $\mathcal{P}$'s become essential , whose size (and correspondingly the virtuality) is defined by their total number depending on Y. If $\mathcal{F}$ is constructed out of such objects, the structure of the internal part of $\mathcal{F}d$ can be nonstationary and becomes more and more black with the growth of energy.

In this paper we discuss these troubles with a $\mathcal{F}d$ structure directely in a parton approach (started and mainly developed by Levin and Ryskin [6]). It has probably a much higher region of applicability, because we don't introduce here auxiliary quasiparticles (like $\mathcal{P}$) and work directly with partons (gluons). But this way is much more complicated, especially for the unification with the existing high energy phenomenology.

It is interesting that in the parton language we meet the close problems with the structure of $\mathcal{F}d$. When we restrict ourselves with only soft partons it is too difficult to produce the black $\mathcal{F}d$, and this leads to troubles, that we discuss from a slightly different point of view in section 2 .

In next sections we try to discuss a part of these problems. Because we expect that for $\mathcal{F}d$ the dense murtiparticle states are most essential we will not consider directely the Fock space vectors for partons, but only the corresponding qusiclassical densities and probabilities.
The outline of this paper is the following.

In **Section 2** the general restrictions on the structure of $\mathcal{F}d$ are considered, that follow from the requirement of the longitudinal Lorentz (boost) invariance (frame independence) of calculations of the ($\mathcal{F}d$ x $\mathcal{F}d$) collisions cross-sections.

In **Section 3** we describe the BFKL inspired parton model for $\mathcal{F}d$ giving the distributions of hard partons with various virtualities in $\mathcal{F}d$ and their variation with Y.

In **Section 4** we consider the ($\mathcal{F}d$ x $\mathcal{F}d$) collision in such a hard parton model and estimate the transparency and their dependencies on the longitudinal Lorentz system.

In **Section 5** we consider a regge model where such a hard $\mathcal{F}$ behavior can arise and where the hard part $\mathcal{P}$ is included in BFKL like manner.

In **Section 6** we mention a number of questions related to the $\mathcal{F}$ type behavior - like of the possible appearance of the $\mathcal{F}$ embryo in existing experimental data; to the similarity of $\mathcal{F}$ x$\mathcal{F}$ collisions with the heavy AxA collisions ; and to the question - if there is any limit from above on the $\mathcal{F}$ -like behavior, possibility connected with gravitational interactions.

2. Unitarity in t-channel, longitudinal Lorentz invariance and the $\mathcal{F}$ limit

The general limitations on the structure of the $\mathcal{F}$ behavior coming from the S-channel unitarity are in some sense trivial and direct - one must "only" carefully calculate all parton currents and exclude multiple counting of cross-sections of the same processes. In particular, the cross-sections of subprocesses must be smaller then the cross-sections of the main processes like $\sigma(\textit{diffraction generation}) < \sigma_{tot}$.

The possible restrictions on the $\mathcal{F}$ behavior coming from the t-unitarity are much more complicated, because one must continue these conditions from t-channel, and this is a nontrivial problem in the parton picture. For $\mathcal{F}$ type behavior the most essential are the n-particle states in t-channel with $n \to \infty$ when $s \to \infty$. And it is in fact unknown how to take the unitarity restrictions on them into account. In the reggeon field theory(RFT) all t-unitarity conditions are automatically fulfilled, when we sum all diagrams. But at high reggeon density the RFT is unapplicable. And in the parton picture, when we directly consider an interaction of stationary hadron Fock states and then have no limitations on the value of parton density, there is no direct way how to control possible restrictions coming from the t-unitarity.

Here we use some approximate conditions which can be partially equivalent to the mean form of the t-unitarity for multiparticle amplitudes. It was proposed in [12], that the t-unitarity restrictions would be equivalent on average to the longitudinal Lorentz (boost) invariance of all cross-sections, calculated in a parton picture. So if we calculate some cross-section using the partonic wave functions $\psi(p)$ and $\psi(p_b)$ of

fast colliding hadrons with momenta p_a, p_b then we expect that this cross-section must be the same in all longitudinal Lorentz frames - that is if we calculate the cross-sections using $\psi(L(\vartheta)p_a)$ and $\psi(L^{-1}(\vartheta)p_b)$, where $L(\vartheta)$ is a longitudinal boost. Remember, that in a parton picture such boosts $L(\vartheta)$ act on hadrons Fock state very nontrivial, changing the number of wee partons, etc. Probably such conditions should be imposed only on the dominant in $1/Y$ (or, maybe, in $\exp(-Y)$?) terms in cross-sections, because for the nonrelativistic interaction the restrictions from the t-channel must be absent.

No strong arguments for such general propositions are known to the author. But, firstly, it is absolutely natural by itself in the parton picture that the calculations of cross-sections, if correct, must give a frame independent answer. And secondly, such a proposition can also be confirmed in RFT if we give the partonic interpretation to reggeon diagrams, by t-cutting them at various intermediate rapidities, as if we calculate various multiparticle inclusive cross-sections. Here we only mention an idea of the construction.

Consider all reggeon diagrams giving a contribution to a total cross-section. If we fix a longitudinal Lorentz frame, such that colliding particles have rapidities y and $Y - y$ - then the cutting of all diagrams at this y can be interpreted as a calculation of the total cross-section (and also various inclusive cross-sections) in a parton picture - and different diagrams in Fig.1 give various contributions, including screenings, to this cross-section. When we go from one section to another $y \to y + \vartheta$, then their partonic contributions to

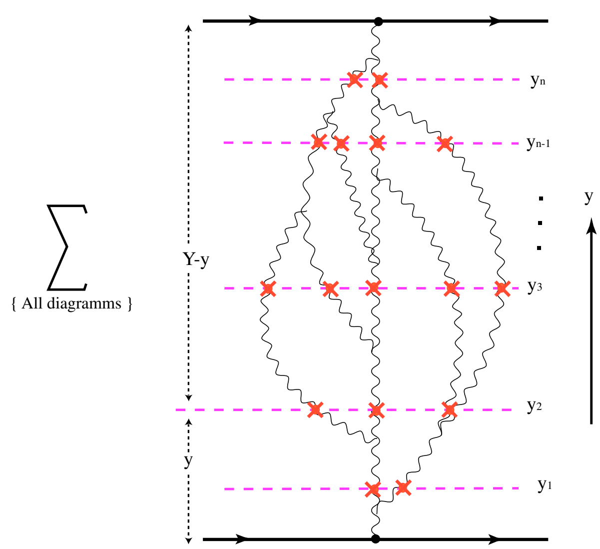

Figure 1: The t-chanel sections y_i of complicated reggeon diagrams. From one side they can be considered as representing the calculation of the cross-sections, using the parton wave functions in various longitudinal frames y_i. From another side - as a sequense of events in the t-channel evolution of a pomeron system along the "time" y

σ_{tot} (and other cross-sections) coming from various diagrams change, but their sum must be the same. The transition between these t-intermediate sequences of the states can be reached by a longitudinal boost. So we move through a sequence of the intermediate states in t-channel, and expect the self-consistency of the answers. Evidently, it is the t-unitarity that guaranties such a consistency.

Let's apply this requirement of the frame independence to the calculation of total inelastic cross-section in the parton approach [3]. Firstly consider the collision of two partonic clouds that are in a state of a rare gas. This is the case normally described by the reggeon diagrams, that, by their construction, include t-unitarity requirements, so here we probably must not meet any problems. Let the mean number of partons in colliding hadrons be $n(y)$, $n(Y - y)$; and the mean transverse radii of regions occupied by these partons

[3]In fact it is known for a long time that in a parton model we can meet some problems with the longitudinal frame dependence. This happens also when we try to interpret in parton terms most pomeron models, that are by their construction frame independent, like the week coupling pomeron [13], with their asymptotic universality of total cross-sections. Similar phenomena appear in a collision of two dense (color-dipole) chains, corresponding to the parton representation of BFKL near the saturation limit [14]. Probably in all such cases the frame independence can only be restored in a rather complicated way, when there is a compensation between processes with very far configurations, as is seen from s-chanel. Or this is at all impossible and signals about a violation of the t-unitarity.

are $R(y)$, $R(Y-y)$ respectively. Then the total inelastic cross-section can be represented as:

$$\sigma_{in}(Y) = \sigma_0\ n(y)\ n(Y-y)\ -\ a_2\ \sigma_0^2\ n^2(y)\ n^2(Y-y)/\Big(\ R^2(y) + R^2(Y-y)\ \Big)\ + ...\ , \tag{1.13}$$

where σ_0 is the parton-parton cross-section, $a_2 \sim 1$. The first term in (1.13) corresponds to a collision of at list one pair of partons. The next terms describe corrections from screening and multipole collisions. For rare parton gas one can in first approximation neglect multiple collisions and screening, that is to leave only the first term in (1.13). Then, from the requirement of the independence of $\sigma_0 n(y) n(Y-y)$ on y it follows the unique solution for $n(y) = n_0 \exp(\Delta_0 y)$ with some constants n_0, Δ_0. It corresponds to a pole in the complex angular momentum plane (and not a cut!) - this condition usually follows in a relativistic Regge approach only from the t-unitarity. Moreover, if we write the cross-section in Eq.(1.13) with definite impact parameter $x_\perp$, then from the frame independence of the $\sigma_{in}(Y, x_\perp)$ the function $n(y, x_\perp)$ is almost completely fixed at $y \to \infty$ (See Eq.(1.22)).

Now let us consider the opposite limiting case of colliding parton clouds, when the parton density is very high and partons fill a transverse disk with the radius $R(y)$. Then the total inelastic cross-section can be determined from purely geometrical conditions - it is defined by the area of an impact parameter space, corresponding to the overlapping of the colliding disks :

$$\sigma_{in}(Y)\ =\ (1-T)\cdot\pi\ \Big(R(y) + R(Y-y)\Big)^2 \tag{1.14}$$

Here $T \ll 1$ is the local transparency of the disks that in general can depend on y, $Y-y$. Now, if $T = const$(y) or can be neglected, from the condition of the independence of the right hand side of Eq.(1.14) from y it evidently follows the unique solution for $R(y) = a\cdot y + b$. So in this case we immediately come directly to the $\mathcal{F}$ behavior of cross-sections.

There can be two main types of corrections to the $\sigma \sim Y^2$ asymptotic.

a) Corrections to σ_{in} resulting from interactions on the spread borders of the disks. Asymptotically at $Y \to \infty$ these corrections are of the order $\sim Y$.

b) Corrections coming from the refining of the value of the transparency T, because in general one can expect that the disk is gray, $T \neq 0$ and varies with y. These corrections can be $\sim Y^2\ \delta T$ in general.

Border type corrections of type a) are connected to the diffraction generation. In this article we will not discuss them and concentrate on processes of type b) taking place in the interior parts of the colliding disks when impact parameter $B\ \leq\ R(y) + R(Y-y)$.

If the parton structure of the $\mathcal{F}d$ at $Y \to \infty$ is mainly generated by the soft processes (probably mostly nonperturbative), and if this $\mathcal{F}d$ has a finite (not a growing with Y) longitudinal thickness, then it is natural to expect that the $\mathcal{F}d$ is gray (and not a black one). That is the transverse local parton density inside the $\mathcal{F}d$ is asymptoticaly finite and doesn't change with Y, and as a result the value of the soft transparency T, entering (1.14), doesn't decrease. This last is especially evident when we move to the laboratory frame of one of the colliding particles. In this case $y \sim 1$, and the fast disk with the radius $R(Y)$ collides with a standing hadron, containing now one-two partons. And then we must expect that there is finite probability that the target hadron can tunnel through such a $\mathcal{F}d$ without interaction (this probability is a definition of the transparency T).

But then we come to a contradictory situation, because in the center-of-mass frame (c.m.) one can expect that the transparency of two disks at the same B can be much less - for example of the order $\exp(-Y^2)$ - because now (on average) more partons interact [4]. To discuss this question more carefully one should consider the possible variation of T with Y and take into account all essential parton configurations, corresponding to the $\mathcal{F}d$, and also these ones that are very far from the mean one.

In general the transparency in a high energy interaction of particles a and b can be expressed as

$$T\ =\ \sum_{i,j} w_i^{(a)} w_j^{(b)}\ \tau_{ij}\quad , \tag{1.15}$$

where we sum over all parton configurations of a and b . In (1.15) $w_i^{(a)}$ and $w_j^{(b)}$ are the probabilities of these configurations, and τ_{ij} - the corresponding transparency in a $|i> * |j>$ colliding state. One can expect

[4]A problem of the same type can take place already in 2 dimensions, where $D_\perp = 0$, if parameters describing the parton fusion (like r_3) are taken very small - they are external parameters for reggeon diagrams. In this case in the lab.frame we have ~ 1 parton collision with a parton cloud of another hadron, and in the c.m.frame $\sim (\Delta/r_3) * (\Delta/r_3)$parton collisions. For $D_\perp = 0$ BFKL color dipol chains there is a similar phenomenon [14].

that for given many parton configurations i j these transparencies are Poisson-like $\tau_{ij} \sim \exp(-cN_{ij})$, where N_{ij} is the mean number of parton collisions in a $|i> * |j>$ scattering. The states with the maximum parton amplitude (probabilities w_i) in (1.15) contain $\mathcal{F}d$, but the corresponding τ_{ij} can be too small, and the main contribution in T can originate from a configurations very far from $\mathcal{F}d$. But firstly we will not take into account such a rare configuration and consider only that being close to the mean one.

Start with the simplest model of $\mathcal{F}d$: it has the radius $R(y) = a \cdot y + b$ and it is filled with partons with the saturated transverse density ρ that is $const(y)$ (in QCD one can expect that $\rho \sim \Lambda^4 \cdot f_\rho((1/\alpha_c))$ inside the disk, and changes to zero on the border. The saturated parton system with high density behaves locally (in $x_\perp$) like a liquid. And the fluctuation of the density of partons is low, because the derivations from the mean value are locally compensated by the partons splinting or fusion. Consider a collision of disks at transverse distances $B \leq R(y)$, and in longitudinally boosted Lorentz system in that disks have rapidities y, $Y - y$ correspondingly. In this case the transparency $T(Y, b)$ concedes with a probability for two disks to go one through another without interaction. If we take into account only the pair interactions between partons with the cross-section σ_0, then it is simple to estimate this probability:

$$T \sim \exp(-\sigma_0 \cdot S_{12} \cdot \rho_\perp^2) \quad , \tag{1.16}$$

where $S_{12}(Y, y, B)$ is the transverse area of disks intersections. The area S_{12} variates with y and B . It is ~ 1 in lab. frame, and $S_{12} \sim Y^2$ in c.m.frame - and as result T is also strongly dependent from y. It is

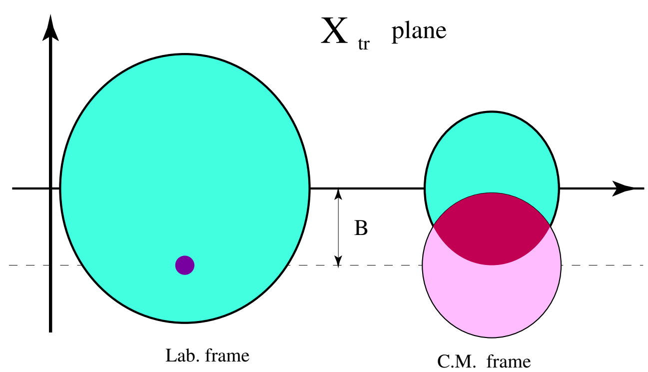

Figure 2: Two $\mathcal{F}d$ intersection in various Lorentz frames at same B. Laboratory (Lab) frame and the frame close to the Center-of-mass frame of a colliding particles (CM) are shown.

evident why it happens. In lab.frame only one-two partons must penetrate through the disk, and in the c.m.frame all partons in area S_{ab} from one disk must independently penetrate through the same layer of partons from another disk - the last probability is exponentially small in comparison with the first one. The simplifications that we made during this consideration like filling disk with parton gas and the pair interactions between partons are not very essential for this general conclusion, because all strong but short range corrections will probably only renormalize parameters entering (1.16), and don't change y dependence of T. For interaction of two gray $\mathcal{F}d$, if the mean configurations dominate (in T), one can only expect, as a generalization of (1.16), the dependence:

$$T \sim \exp(-c_0 \cdot f_1(Y, y, B)) \ , \tag{1.17}$$

where $f_1(Y, y, B)$ is the inclusive spectrum of primary partons at definite B.
What explanations for this situation can be presented, and how can we avoid such a trouble ?

- It can be considered as a symptom that one must leave the $\mathcal{F}$ regime as a too restrictive one for realistic field theories with finite range interactions, like a case with asymptotically constant cross-sections (week coupling [13]). They are very similar in many respects. And then the real asymptotics can probably correspond to a strong coupling regime with $\sigma_{in} \sim Y^c$, $0 < c < 2$. But here we will not discuss this possibility further on.

- Suppose that all $\mathcal{F}d$ are gray with the finite and Y-independent transparency in the lab. system. But at the same time suppose also that these partons are so finely correlated in the whole $\mathcal{F}d$, that the same finite transparency remains somehow also in other frames. Then, if such strongly correlated configurations dominate, the Eq.1.16 becomes unapplicable. One cannot simply exclude this variant in general, but it remains unclear how such a $\mathcal{F}d$ can be constructed from massive partons with a finite range of interactions. As an example for this gray $\mathcal{F}d$ we can mention the critical $\mathcal{F}d$, discussed in the Introduction. It is hard to believe that such versions can be considered seriously without some additional reasons.

- Search the rare parton configurations that contain a relatively small number of partons and so can give the maximal contribution to the transparency. Such configurations can arise from big fluctuations in the initial state of the parton cascade. To increase the transparency in frames close to c.m. one can ask for an additional parton component $|\varphi>$ for a fast hadron that doesn't contain $\mathcal{F}d$ at all and interacts slowly (or doesn't interact at all). Schematically : $\psi(p_a \to \infty) \simeq a_f|disk\ \ > + \ a_i|\ \varphi\ \ >$, where a_i is the amplitude of the $|\varphi>$ state. It corresponds to the limiting case of Eq.(1.15), where we have kept only two states. The probability for a hadron not to have a $\mathcal{F}d$ (or to have it with a small radius less then B), is connected with a_i like $w(y) \sim |a_i|^2$. In this case the expression for the transparency can be generalized to :

$$\begin{aligned} T \quad \sim \quad & \exp(-c_0 \cdot f_1(Y,y,B)) \cdot \big(1 - w(y)\big) \cdot \big(1 - w(Y-y)\big) + \\ & \tau_{\varphi d} \cdot \big(w(y) + w(Y-y)\big) + \\ & \tau_{\varphi\varphi} \cdot w(Y-y)) \cdot w(y)\ , \end{aligned} \tag{1.18}$$

where for soft gray $\mathcal{F}$ transparencies $\tau_{\varphi d}$ and $\tau_{\varphi\varphi}$ are finite and y independent. Then at first sight one can expect that the last term in (1.18), coming from the $|\varphi> \cdot |\varphi>$ component, can dominate and so can make T boost invariant. But it is possible only if $w(y)$ is approximately constant for high y. And, at the same time, there are no known mechanisms that can give such a constant w. Various estimates of $w(y)$ lead to a decreasing function of the type $w(y) \sim \exp(-\gamma \cdot y)$. It coresponds to the choice at every rapidity stage of such an evolution direction, that doesn't increase the parton number. Such w leads to the expression

$$T(Y,y) \ \sim\ \tau_{\varphi d} \cdot \big(\ e^{-\gamma\ (Y-y)}\ +\ e^{-\gamma\ y}\ \big)\ , \tag{1.19}$$

corresponding to the collision of the rare state $|\ \varphi\ >$ with $|\ \mathcal{F}d\ \ >$, and such T is y dependent. Therefor on this way the soft $\mathcal{F}d$ can also not be cured.

- And finally one can consider such models of $\mathcal{F}$ that the full parton density $\rho_\perp$ inside the disk doesn't reach saturation at a given B, but continues to grow with y and varies with B. In this case we can hope to compensate the y dependence of S_{12} in Eq.(1.16), or make the transparency of $\mathcal{F}d$ for one parton (in lab.frame) so small ($\mathcal{F}d$ becomes more and more black with y) that the configurations $|\ \varphi > \cdot\ |\ \varphi >$ and not $|\ \mathcal{F}d\ > \cdot\ |\ \varphi >$ become dominant in T.

But firstly consider what happens with T in the parton approach corresponding to $\mathcal{F}$ in the eiconal model with the supercritical $\mathcal{P}$, mentioned in the Introduction. Such a $\mathcal{P}$ corresponds to a chain of splitting partons, with the simplest regge type transverse distribution $\rho_\perp(y,B) \sim y^{-1} \cdot \exp(\Delta\ y - B^2/4\pi\alpha' y)$. This density $\rho_\perp(y,b)$ coinsides with the Green function of $\mathcal{P}$. The multiple $\mathcal{P}$ exchange takes into account the screening in the process of an interaction of two hadrons so that the probability of the interaction at given B doesn't exceed 1. It is essential that because in this approximation $3\mathcal{P}$ vertex (and also higher vertices) is set to zero, partons don't glue despite their density exponentially grows. We must here slightly refine the formula (1.16), because now the density depends on the transverse position :

$$T \sim \exp\Big(\ -\sigma_0 \cdot \int d^2b \cdot \rho_\perp(y,b) \cdot \rho_\perp(Y-y, B-b)\ \Big) \tag{1.20}$$

Then we see that the integral in the exponent in (1.20) is $\sigma_0 \cdot \rho_\perp(Y,b)$ and doesn't depend on y, and as a result the contribution to the transparency also from the $\mathcal{F}d$ configuration is boost-invariant. The infinite growth of the soft parton density was here essential to go to the consistent $\mathcal{F}$ type behavior.

Note that here, for rare configurations, that are far from the main one, we also have a frame independent behavior of T. The maximum of T is reached here when both colliding parton clouds fluctuate to rare $\sim$(one soft parton) states. And it gives

$$T(Y,y) \ \sim \ e^{-\gamma\ y} \cdot e^{-\gamma\ (Y-y)} \sim e^{-\gamma\ Y} \ , \tag{1.21}$$

that corresponds to the last term in Eq.(1.18) The difference between (1.21) and the gray disk case (1.19) is the following: here, to minimize T, we have a symmetrical fluctuation, where, for a gray disk case, the maximum of T is reached on an unsymmetrical configuration.

This example raises a natural question. Can one somehow generalize this simple eiconal mechanism to come to $\mathcal{F}d$ with a finite density and a frame invariant T ? To understand this, let us try to find the general $\rho_\perp(y,b)$ for that the integral in (1.20) is y-independent, and σ_0 doesn't depend on y and B. The solution of the corresponding equation for $\rho_\perp$ can be represented as:

$$\rho_\perp(y,b) \sim \int d^2k \cdot f_1(k) \cdot \exp(ikb + yf_2(k)) \ , \tag{1.22}$$

where f_1, f_2 are arbitrary functions of k. For $y \to \infty$ the integral in (1.22) can be taken by the steepest decent method, so that only the neighborhoods of zeros of $\partial f_2(k)/\partial k$ are essential. Then from the positivity of $\rho_\perp$ it follows that f_2 is positive and so the dominant contribution must come from the region $k \sim 0$, otherwise $\rho_\perp(y,b)$ will oscillate in b. So in the essential region $f_2(k) \simeq c_1 - c_2k^2$, and in fact we return to the pole-like form of $\mathcal{P}$ [5].

Here we also describe briefly mechanisms, responsible for the transparency of the gray $\mathcal{F}d$ in the regime of the strong coupling mentioned in the Introduction. For such a $\mathcal{F}d$ all diffractive and other processes with a high derivation of density from the mean value are of the same order in Y as a $\sigma_{tot} \ \sim \ Y^2$. So fluctuations in individual events at $Y \to \infty$ must be very high. For example, the inclusive spectra for rapidity gaps of length y_1 can be found from cutting the self energy diagram of the type $\left[\ G\ (\Gamma_3 GG\Gamma_3)\ G\ \right]$, where G and Γ_3 are given by (1.9). From here the mean multiplicity of such gaps of length $> y_1$ can be estimated as:

$$n_g(> y_1) \ \sim \ Y/y_1$$

For $y_1 \sim Y$ this in fact gives the value of the cross-section of dif.generation ($\sim \sigma_{tot}$), and for small y_1 we have $n_g(y_1 \sim 1) \ \sim \ Y$. This shows that the average parton state looks like a gas(liquid ?) in the critical point, with a high fluctuation of the relative density of the order ~ 1 on a scale of the full system size.
It can then explain how the finite transparency of two $\mathcal{F}d$'s in the center-of-mass frame can be achieved. Due to large fluctuations in such a parton system there is an y-independent probability of order ~ 1 that the soft partons of one $\mathcal{F}d$ collide with a state of other $\mathcal{F}d$, containing a large rapidity hall (gap) with no soft partons in the same interval of rapidity. Then disks can freely move one through another.
But such a critical $\mathcal{F}d$ construction needs a fine tuning of an infinite number of parameters and looks too artificial - we will not consider it further.
So in fact there remain two asymptoticaly different possibilities :

1) If $\mathcal{F}d$ consists only of soft partons, their transverse density must grow with Y, for example like $\sim \exp \Delta y$ or slower. In terms of RFT with the $\mathcal{L}$ given by the Eq.(1.8) it corresponds to such a parameter choise, that the ψ system has no ground state and ψ continues to grow with the 'time' y, inside the $\mathcal{F}$ bubble. If the longitudinal size $L_z(Y)$ of this $\mathcal{F}d$, where the partons with lowest momenta are distributed, is finite $\sim 1/m$ and doesn't grow with y asymptotically (as it is usually believed), then this version must be probably also abandoned. But if $L_z(Y)$ grows with Y, so that at least the 3-dimensional density of soft partons remains constant, the situation can change. In fact, already the very slow growth of $\rho_\perp(y,b) \sim \log^{1+a} y, \ a > 0$ is quite enough to make the contribution from $|\varphi \cdot \varphi>$ to T more than from $|\ \mathcal{F}d \cdot \varphi >$, and this gives the frame independent answer (1.21).

It was proposed in [15] that the mean longitudinal size of the region, filled by partons with the energy ϵ, can grow like $L(E,\epsilon) \sim E/\epsilon^2$, that gives $L_z(Y) \sim L(E, \epsilon \sim m) \sim \exp(y) \sim E/m^2$. Then in such a disk

[5] For a collision at large $B \simeq aY$, that corresponds to the collision of $\mathcal{F}d$'s with their borders, the first line in (1.18) becomes dominant and gives $T \sim 1$. Here to realize the frame independence of T on must carefully adjust the soft $\rho_\perp(y,b)$ and the dependence of the $\mathcal{F}d$'s radius on Y, to slightly separate the borders of two colliding $\mathcal{F}d$'s in c.m. frame in comparison to the lab.frame. The nonlinear dependence of $R(Y) \sim aY - c\log Y$ helps in the case of dense $\mathcal{F}d$ with the sharp border. It can be considered as coming from the surface tension on the border of the growing new phase bubble: $\partial R/\partial y \sim const - 1/R$. But if the border of $\mathcal{F}d$ is diffuse, then the parton density $\rho_\perp(y,b)$ must perhaps be close to Gauss form, like in (1.22). This note concerns the most models of $\mathcal{F}d$, because the $\mathcal{F}d$'s border is always expected to be soft and 'gray'.

(now looking more like a tube) one can arrange partons with a finite density (in the volume) and so make an almost "black" soft object with the low transparency. This version is not realised in a perturbation theory - for ladder diagrams one can simply estimate that $L(E,\epsilon)$ is always $\sim 1/\epsilon$. But for a dense parton gas, the growing "pressure" can push "additional" partons in the longitudinal direction and lead to the growth of $L_z(Y)$. It is very complicated to convince if this possibility can be realized in a field theory, partially because we have no soft field theories in 4 dimensions, and at the same time there is no supercritical perturbative Pomeron in lower dimensions. [6].

2) The other way is also to include an increase of the partons density with y, but now we don't put them in the longitudinally elongated part of $\mathcal{F}d$ - rather than force them to increase their mean transverse momenta and virtuality. This last one is very natural in a renormalizable field theory like QCD - even this behavior shows the solutions of the BFKL equations, in that the partons spread in transverse momenta with the growth of y. So one can expect that the mean transverse momenta of partons will grow with y. As a result the mean cross-section of their interaction σ_0 will effectively depend on y and $Y-y$. This changes the expression (1.18) for the transparency and enlarges the classes of function $\rho(y,b)$ for that T is frame independent. And, like in the previous case, for a very dense $\mathcal{F}d$ the $|\varphi\cdot\varphi>$ component will dominate in the expression for T. This question will be discussed in the next section. The possibility that the high $k_\perp$ can dominate in saturated parton configurations was mentioned in an number of papers, starting from [6], and is the most natural way for explaining various properties of $\mathcal{F}d$ in QCD including the transparency.

3. Partonic structure of a hard $\mathcal{F}$ disk in QCD.

In this section we discuss the parton structure of $\mathcal{F}$, inspired by various perturbative generalizations of the BFKL approach. What we firstly need is an approximate qualitative picture that shows how by growing of Y the $\mathcal{F}d$ is filled with partons-gluons at various virtulities (transverse momenta). For this purpose we supplement the BFKL equation with terms describing a gluon fusion [17] and also include in it the running coupling constant. To simplify considerations we also split, in the kernel of the BFKL equation (in the manner presented in [19]), the diffusion processes in $u = \log(k_\perp)$ and express them in the differential (local in u) form, and the DGLAP processes, for that one can use a trivial (only the singular part) kernel. Such a model can be represented by the following equation for the gluon density $f(y,u)$:

$$\begin{aligned}
\frac{\partial f(y,u)}{\partial y} \quad &= \quad \delta\cdot\alpha_c(u).f(y,u) + B_c\alpha_c(u)\cdot\frac{\partial^2 f(y,u)}{\partial u^2} + ... \\
&\quad - \lambda_2 e^{-u}\cdot\alpha_c^2(u)\cdot f^2(y,u) + ... \\
&\quad + \int_{\sim 0}^{u} du_1\alpha_c(u_{1)}f(y,u_1) + ...
\end{aligned} \tag{3.1}$$

The first line in (3.1) gives the BFKL like evolution in y and u with the running QCD coupling $\alpha_s(u)\simeq 1/(bu+\alpha_0^{-1})$, that is 'freezed' on the value α_0 in the infrared region. The second line corresponds to the fusion of partons - where we have represented (also in the local form) only the first term of these series, describing the transition of two gluons in a single one. The third line represents DGLAP type processes, where a parton can change fast its u on a small interval of rapidity. In Eq.(3.1) we didn't fix explicitly the coefficients ($\delta = 12\log 2/\pi + \alpha_c\ \delta_2\ +\ ...,\ B_c,\ \lambda_2, ...$) - we only extracted the α_c dependencies from them [7].

If we, for a moment, forget about the diffusion of partons over the virtuality u and also about their gluing, then from Eq.(3.1) follows the behavior of the $f(y,u)$:

$$f(y,u)\ \sim\ f_g(y,u) = f_0\int_0^y dy_1\ e^{(y-y_1)\delta_1/u} I_0(2\sqrt{y_1\xi(u)}) \tag{3.2}$$

[6] In this connection it is interesting to note that in the string theory, that is in fact a soft theory on the string scale κ, there can exist a close phenomenon. It was remarked ([16] and later works), that the longitudinal size of the fast string grows like $\sim E/\kappa^2$ due to the inclusion of more and more high frequensity internal harmonics in game. But the natural question - if we can learn from here something about the possible structure of the soft $\mathcal{F}d$ - remains open, because here we necessary must include a gravitational interaction on the same scale, and that corresponds to the theory very far from QCD, and so needs additional considerations (see remarks in section 6).

[7] Note, that the choice of gluons as partons is not an unique one. It is probably more correct to use color dipols [20] as partons - this is more evident and also essential for not meeting the infrared problems in a Fock wave function. But, because we are mostly interested in configurations with high parton numbers, it is probably irrelevant.

where $\delta_1 = \delta/b$, $\xi(u) \sim (12/b)\log(\alpha_c(u_0)/\alpha_c(u))$. The evolution, described by Eq.(3.2) corresponds to the DGLAP jump in u, followed by the density growth from parton splitting. At $y > \hat{y}(u) \simeq u^2\xi(u)/\delta_1^2$ (3.2) simplifies

$$f(y,u) \sim f_0(u)\ \exp(\ \delta_1 \frac{y}{u}\ )\ \ , \tag{3.3}$$

where $f_0(u) \sim \exp(\xi(u)/\delta\alpha_c(u))$. At small $y \ll \hat{y}(u)$ the growth of $f(y,u)$ is more close to the DGLAP type

$$f_g(y,u) = f_0 \sum_{n=0}^{\infty} \left(\frac{y}{\hat{y}(u)}\right)^{n/2} I_n(2\sqrt{y\xi(u)})\ \sim\ \frac{f_0}{(y\xi)^{1/4}} e^{(2\sqrt{y\xi(u)}\ )}\ +\ ... \tag{3.4}$$

When the density of partons with the virtuality u in some part of disk becomes very large then the recombination of gluons becomes essential, and the future growth of $f(y,u)$ with y can stop, and the density comes to the saturation. This is described by nonlinear terms in Eq.(3.1). The corresponding limiting value for gluons with the virtuality u can be estimated as

$$f_{sat}(u) \sim \frac{\delta}{\lambda_2 \alpha_c(u)} \cdot e^u \tag{3.5}$$

At these values of f the equilibrium between splitting of u-partons and their joining is reached. The DGLAP type processes in this region give the small contribution. In the expression (3.5) for f_{sat} only the $\sim f^2$ nonlinear term from (3.1) is taken into account. In fact near the saturation region all nonlinear terms can be of equal importance. It probably will only change the coefficient in (3.5). If neglecting the transport of partons in u in Eq.(3.1), their density evolution is described by the equation

$$\partial f/\partial y \ \ =\ \ V(f) \equiv \alpha_c f \cdot (\delta - \lambda_2 e^{-u}\alpha_c f + \lambda_3 e^{-2u}\alpha_c^2 f^2 - ...\ ) \tag{3.6}$$

The value f_{sat} corresponds to the point, where $V(f_{sat}) = 0$, and because f enters nontrivialy in $V(f)$ only in the combination $\alpha_c e^{-u} f$, it will again lead to (3.5). But if $V(f)$ has no zeros at $f > 0$, then there is no complete saturation. If $V(f)$ freezes at values $\sim e^u$, then, after f reaches values (3.5), a slow universal growth of the parton density is possible with y like

$$f\ \sim\ e^u\ (y - y_0(u))\ , \tag{3.7}$$

where $y_0(u)$ can be $\sim u^2$. The examples of such a V reminding of eiconal or Eq.(1.5) are

$$V(f)\ \sim\ e^u\Big(1 - \exp(-\lambda\alpha_c e^{-u}\ f)\Big) \qquad \text{or} \qquad V(f)\ \sim\ (\delta\alpha_c f)/\Big(1 + \lambda\alpha_c e^{-u}\ f\Big)\quad ,...$$

The possibility of such a behavior of f was discussed in a number of works [21]. It is hard to understand now what a behavior takes really place in QCD at high densities, but for our purposes, connected with the structure of $\mathcal{F}d$, it is not so essential. Probably the best way to approach to this question, and also to the whole parton structure of the $\mathcal{F}$ limit, is to try using the direct color field representation for the $\mathcal{F}d$, in the manner proposed in [22], and then to apply the corresponding longitudinal renorm-groop equations [21], instead of (3.1). We will not try to do this here, but suppose simply that the full saturation of the type (3.5) takes place.

We need also the approach to the evolution of partons in the transverse coordinate for a $x_\perp$-scale large as compared to Λ_c^{-1}. For that we explicitly include in f the third argument $x_\perp$, omitted in Eq.(3.1) [8]. Remember that for the parton evolution, represented by one Regge pole, the corresponding "diffusion" equation for density is:

$$\partial f(y,u,x_\perp)/\partial y\ =\ \Delta(u)\cdot f(y,u,x_\perp) + \alpha'(u)\cdot \partial^2 f(y,u,x_\perp)/\partial x_\perp^2\ \ , \tag{3.8}$$

Here $\Delta(u)$ is the u-dependent intercept, and $\alpha'(u)$ is the u-dependent slope of $\mathcal{P}$ representing the diffusion coefficient (in $x_\perp$) for partons with the virtuality u. The first term at the right hand side of Eq.(3.8) gives the parton splitting and corresponds to the same term in Eq.(3.1) with $\Delta(u) = \delta \cdot \alpha_c(u)$. The second term from r.h.s. of the (3.8) is absent in (3.1), and to combine the two equation we simply add it to Eq.(3.1). We can get information about the order of $\alpha'(u)$ from the t-dependence of $\mathcal{P}$ positions, as it follows from solutions of the BFKL equation with the running coupling; it gives the estimate:

$$\alpha'(u) \sim \alpha_c^2(Q^2)/Q^2\ \sim\ e^{-u}\cdot u^{-2}\ \ , \tag{3.9}$$

[8] This is the result of simplifications, used in a standart main-log's approaches to equations of type (3.1).

One point from (3.9) is essential for later estimates - that the slope $\alpha'(u)$ rapidly decreases with u. As a result we only need the value of $\alpha'(u)$ at small(minimal) u to find how fast $\mathcal{F}d$ expands with y. Just here the nonperturbative QCD contributions are the maximal ones. But one can hope that the nonperturbative effect, if correctly included, will not change the general structure of equation (3.1) with a frozen α_s, as long as we use gluons as partons. So we use simply some (phenomenological) value of the α' for the soft $x_\perp$ parton diffusion. Such a parameter enters into the calculation of the velocity of the expansion of the soft part of the $\mathcal{F}d$; and as a result it follows from (3.1, 3.8):

$$R_{soft}(y) = r_0 \cdot y \tag{3.10}$$

This value of r_0 fixes the transverse scale in our problem - so that later all other quantities can be measured in such r_0 units. Therefore, to simplify all expressions, we will use r_0 units for transverse distances, so always use $r = x_\perp/r_0$, instead of $x_\perp$. Hence at the border of the $\mathcal{F}d$ we have $r = y$.

All partons belonging to the fast particle fill on average the cone $\mathcal{FC}$ defined by conditions ($r^2 < y^2$) in the $(\mathbf{x}_\perp, y)$ space. Their section at $y = Y$ gives the soft $\mathcal{F}d$, as seen in the frame where the colliding particle momenta are $\sim e^Y$. Inside this $\mathcal{FC}$ the soft partons come to the limiting (saturated) density. Meanwhile we neglect the spreading of the border of this $\mathcal{FC}$ - we consider it later.

The next question we discuss is: how are the hard partons with the virtuality u distributed inside the main soft $\mathcal{FC}$? Let's choose some point (r_1, y_1) inside the $\mathcal{FC}$ and consider how the partons with the virtuality u arise near it. There are various mechanisms included in equations (3.1) that are responsible for that. Or, in other words, various paths in (r, y) spaces, by that partons evolve to (r_1, y_1) point, starting their evolution from the top of the $\mathcal{FC}$. One can easily show that the main contribution comes from such a path, illustrated in Fig.3 : the partons start to evolve from the soft state with $u \sim 1$ and "move" not

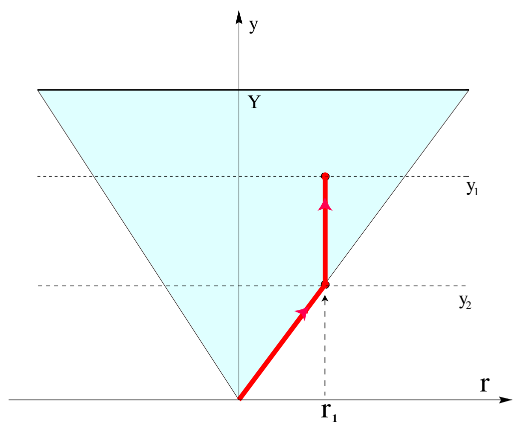

Figure 3: The main trajectories of parton evolution inside $\mathcal{FC}$, that contributes to the quasiclassical parton dencity.

changing their virtuality from $(r = 0, y = 0)$ to the point $(r = r_1, y_2 = r_1)$, located near the border of the $\mathcal{FC}$, then, staying in this point, they jump in u to some higher value u_1, and finally evolve in y at a fixed r to the point (r_1, y_1), by growing of their density with y like (3.2). This shows that the mean density of these u partons can be in two "states". If u are large enough and the u-partons have had no "time" to reach the saturation limit, then their density is defined by the Eq(3.2) with $y \to y_1 - r_1$. From the other side, if u-partons had enough long path in y to reach the corresponding saturation limit, then their density would be freezed at values given by the Eq.(3.5). These two regimes coincide at the values

$$u \simeq u_s = \sqrt{\delta_1 (y - r)} \qquad \text{or} \qquad r = y - \delta_1^{-1} u^2 \tag{3.11}$$

The main asimptotic property of such hard $\mathcal{F}d$ is that partons for all virtualities less than $u_s(y, r)$ are in a saturation phase, and for virtualities greater than u_s the corresponding parton densities continue to grow with y. One can combine this all in one approximate expression :

$$\begin{aligned} f(y, u, r) \; = \; & \theta(y - r - \delta_1^{-1} u^2) \cdot f_{sat}(u) \quad + \\ & \theta(r - y + \delta_1^{-1} u^2) \cdot f_g(y, u) \end{aligned} \tag{3.12}$$

where f_{sat} is given by (3.5), and $f_g(y,u)$ by (3.2).
It is interesting to reinterpret this expression considering the distribution of parton density at given u as a function of the transverse distance r. We have accepted, that at the small u this distribution is approximately a disk with the radius $R = y$, the soft parton density inside is $f_{sat}(0) \sim 1$, and the width of the border in r is also ~ 1. At higher values of u the expression (3.12) also represents a θ-like disk - the first term in the right hand side corresponds to its interior, and the second to the smeared border. Such a u-disk has a radius $R(u) = y - u^2$, the density inside $\sim f_{sat}(u)$, and the border spread in r by $\sim u/\delta_1$, the form of that is given by the last term in Eq.(3.12).
The second term in Eq.(3.12) also can be considered as describing the distribution (over u) of parton densities from the virtualities $u > \sqrt{y}$ up to the values $u \sim y$, when their densities only grow and are far from a saturation.

Therefore we can represent the parton structure in the $\mathcal{F}$ limit as a system of enclosed disks, with various virtualities u and radii $R(u) = Y - u^2$, decreasing with the growth of u and with higher and higher densities $\sim e^u$. All these u-disks are enclosed in the main soft $\mathcal{F}d$, and expand with the same velocity in y. The u-disks are gray, but their transparency decreases when u grows. The total parton density in r is given by the sum of densities of individual u disks (see Fig.4), and can be approximately represented by the enveloping curve in Fig.4 :

$$\rho(r,Y) \simeq \sum_{u\sim 1}^{\sqrt{\delta_1(Y-r)}} f_{sat}(u)\ \theta(Y - \delta_1^{-1}u^2 - r)\ \sim\ e^{\sqrt{\delta_1(Y-r)}} \tag{3.13}$$

When Y grows then in the middle of the main $\mathcal{F}d$ a new u-disk is created with the virtuality $\sim \sqrt{y}$ and then it expands like all other disks with smaller virtualities. This picture represents qualitatively the

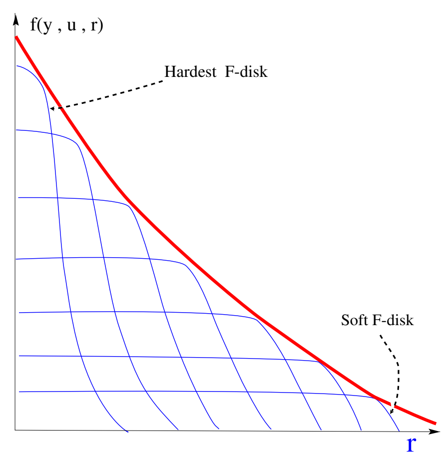

Figure 4: Tower of $\mathcal{F}$ -disks with a growing virtualities. The enveloping curve corresponds to Eq.(3.13)

structure of a partonic wave function of the $\mathcal{F}$ limit, that is in essential part a quasiclassical one, because the fluctuations of all densities are small compared to the mean values. Evidently such u-disks cannot be separated one from another, and their introduction is used here only to make the presentation more visual.

At the end of this section let us discuss what are the parameters of $\mathcal{F}d$ accompanying fast but very small object like "onia" with the size $R_0 \ll \Lambda_{QCD}^{-1}$; it can be also a virtual DIS photon that has a size $R_0 \sim Q^{-1}$. One can expect that the BFKL-like gluon chain is directly attached to such a small object, with the starting virtuality $u_R \sim \log R_0^{-2}$. When u_R is large there is in fact no transverse motion of chain partons in the $x_\perp$ plane. But during the evolution of this parton chain in rapidity, when going to smaller and smaller momenta, its density grows like (3.2) and finally can reach the saturation limit corresponding to the virtuality u_R. Also during the BFKL like evolution in rapidity the set of vitrualities presented in such a gluon(parton) chain expands by the "diffusion" in u. The virtualities higher than u_R are also reached by the "DGLAP jumps" in u (as described by the integral term in (3.1)) and after that their densities grow as in (3.2). The spreading in u is a such one that the soft virtualities are reached with a probability of order

of unity when

$$y \; > \; y_e(u_R) \sim \alpha_s^{-1} \; (u_R) u_R^2 \; \simeq \; c_e \; u_R^3 \; , \tag{3.14}$$

where $c_e \sim 10^{-2}$. After that the soft cloud around the u_0 particle is created, and then the soft critical density is reached, the soft $\mathcal{F}d$ starts to grow in $x_\perp$. Then it produces more hard $\mathcal{F}$ disks, and so on, as described before for soft ends. Thus for a small highly virtual object all $\mathcal{F}$ picture is displaced in y by $c_e \; u_R^3$, so that asymptotically the radius of the soft $\mathcal{F}d$ is given by:

$$R_{soft}(y, u_R) = r_0 \; y - c_e u_R^3 \tag{3.15}$$

More hard $\mathcal{F}$ disks with virtualities u are enclosed in R_{soft} with the same shift by $c_e u_R^3$ of their radii.

The picture of $\mathcal{F}$ described in this section and based on the Eq.(3.1) corresponds to average (dominant) configurations. Because such components of $\mathcal{F}$ contain many particles in dence saturated states, the system is close to a classical one in some respects, and, as a result, fluctuations around such mean $\mathcal{F}d$ are small - they are probably Gaussian ones for a not too large deviation.

But to treat the transparency of $\mathcal{F}d$ we need also probabilities of large fluctuations. They originate, as in all cascading processes, mainly from a fluctuation in initial stages of cascading (small y), followed then by the same type of the fluctuations on every step. In a high energy parton configuration we have $\sim y$ steps of evolution (parton splitting). On every such a step the density grows on average. This leads to exponentially damped in y probabilities of rare configurations. At first we need

$$w_s(y) \;\; \sim \;\; \exp(-\gamma_s y) \tag{3.16}$$

where $\gamma_s \sim \Delta$, that gives the probability that for a fast particle there would be no cascading (or minimal one) at all. This gives the parton states containing ~ 1 soft parton on every rapidity step and interacting with the target with $Const(y)$ cross-section.

The second probability that can be needed for estimates of T corresponds to configurations, in which we have only a "normal" soft gray $\mathcal{F}d$, but hard components of $\mathcal{F}$ are not generated. Because the hard components can start to grow from every region of soft $\mathcal{F}d$, and at every y, we can expect that this probability

$$w_h(y) \;\; \sim \;\; \exp(-\gamma_h y^3) \; , \tag{3.17}$$

where $\gamma_h \sim \alpha_c(k_\perp \sim \text{min. hard scale })$.

Let's note that the probability for the creation of a hole with the area S_h on the hard disk, through that another colliding particle can penetrate is $\sim \exp\left(-\gamma_h y S_h\right)$. This also can be essential only in frames close to the lab.frame of one of the colliding particles.

4. Collision of two hard $\mathcal{F}$ disks.

Now we consider the collision of two such hard $\mathcal{F}$ disks and estimate the transparency in various longitudinal systems. Firstly consider only the parton configurations close to the mean one. In the $(\vec{x_\perp}, y)$ space the $\mathcal{F}d$ * $\mathcal{F}d$ collision with the definite transverse distance B can be represented by the intersection of two cones filled with partons (see the Fig.5).

In this picture the choice of the longitudinal Lorentz frame in that we consider the collision of two $\mathcal{F}d$ corresponds to the sections at fixed y. As it was discussed in the previous section, both cones contain embedded subcones with larger virtualities u. In the process of the collision partons, that are in the region of $\mathcal{F}$ cones intersection (region D on Fig.5), interact, and produced secondary particles (jets -for a high virtuality) fill the region D. Because secondary particles with high $p_\perp$ come mainly from a collision of subcones with a high virtuality - these particles are concentrated near the line **L** passing through the central part of D. And therefore at high Y the main contribution to opacity in average configurations also come from the parton collisions near the line **L**.

With the exponential precision the transparency can be expressed through the parton densities, given by Eq.(3.12) as:

$$T \;\; \sim \;\; \exp\Big(- \Gamma(Y, y, B) \Big) \; , \tag{4.1}$$

where

$$\Gamma(Y, y, B) \;\; \sim \int d^2 x_\perp \int du_1 du_2 \; \sigma(u_1, u_2) \cdot f(y, u_1, |\vec{x}_\perp|) \cdot f(Y - y, u_2, |\vec{B} - \vec{x}_\perp|) \; , \tag{4.2}$$

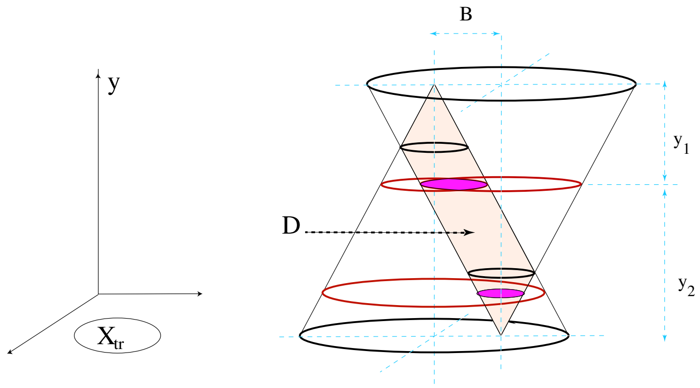

Figure 5: Two intersecting F-cones in $(x_\perp * y)$ space. Their sections at rapidities y_i give the picture of two $\mathcal{F}d$ collisions in the corresponding frames. From the region **D** of two $\mathcal{FC}$ intersection particles are produced.

and

$$\sigma(u_1,u_2) \sim 1/(k_{1\perp}k_{2\perp}) \sim \exp(-(u_1+u_2)/2)$$

is the cross-section for the parton interaction with virtualities u_1, u_2 .

It is instructive to estimate this integral by choosing two u-subdisks that are maximally opaque and catch each other during $\mathcal{F}d$ collision. This, in particular, can show the $((\vec{x_\perp}, y))$-geometry of main inelastic processes during the $\mathcal{F}d$ * $\mathcal{F}d$ collision. Let us fix some y-frame, in that the corresponding rapidities of $\mathcal{F}d_1$ and $\mathcal{F}d_2$ are y_1, $y_2 = Y - y_1$, and the ipmact parameter $B < Y$. Next pick out from these $\mathcal{F}d_i$ the subdisks with such virtualities u_1 and u_2, that these subdisks still overlap, it is the sum of their radii

$$r(u_1) + r(u_2) = Y - u_1^2 - u_2^2 < B \ . \tag{4.3}$$

The area of the overlapping region $S_{12} \sim \int d^2\vec{x}\ \theta\ (|\vec{x}| - y_1 + u_1^2)\ \theta\ (|\vec{x} - B| - y_2 + u_2^2)$ varies with B from 0, when disks only touch each other, to a $\min_i\ [\ \pi\ (y_i - u_i)^2\ ]$ - when they overlap completely at $B = 0$. Also, in a such estimate we can don't take into account the spreading of borders of these u-disks. Then the simplest quasiclassical estimate of their reciprocal transparency $T(u_1, u_2, B)$ is given by :

$$\begin{aligned} T(u_1,u_2,B) &\sim \exp(\ -\Gamma_{u_1,u_2}\ ) \\ \Gamma_{u_1,u_2} &= S_{ab}\cdot\sigma(u_1,u_2)\cdot f(y_1,u_1,r(u_1))\cdot f(y_2,u_2,r(u_2)) \ . \end{aligned} \tag{4.4}$$

In (4.4) the most fast changing combination behaves as :

$$\sigma\cdot f\cdot f \ \sim\ f_{sat}^2 \exp[(u_1+u_2)/2] \ \sim\ \exp[(\sqrt{y_1 - r_1} + \sqrt{y_2 - r_2})/2] \tag{4.5}$$

Therefore the maximum in Γ is reached when these u_i disks touch only each other - that is when $r_1 + r_2 \simeq R$. And this last maximum of $\sigma\cdot f\cdot f$ is reached for disks with radii:

$$r_1 = \frac{B}{2} + \frac{y_1 - y_2}{2}; \qquad r_2 = \frac{B}{2} - \frac{y_1 - y_2}{2} \tag{4.6}$$

at $|y_1 - y_2| < B$ (region D_2 on Fig.6) , and the corresponding virtualities are :

$$u_1 = u_2 = \sqrt{(Y-B)/2} \tag{4.7}$$

It is correct when disks intersect only partly (y-sections in region D). And when one of u-disks is completely inside the another, then (Regions D_1 and D_3):

$$\begin{aligned} r_1 &= B, \quad r_2 \simeq 0, \quad u_1 = \sqrt{B}, \quad u_2 = \sqrt{y_1 - B}, \quad y_1 > R + y_2 \\ r_1 &= 0, \quad r_2 \simeq R, \quad u_2 = \sqrt{B}, \quad u_1 = \sqrt{y_2 - B}, \quad y_2 > R + y_1 \end{aligned} \tag{4.8}$$

We see that the main contribution to the Γ comes from a thin region (tube) surrounding the line **L** (see Fig.6) that passes through the middle of the region D. It is the region, in that the maximum number

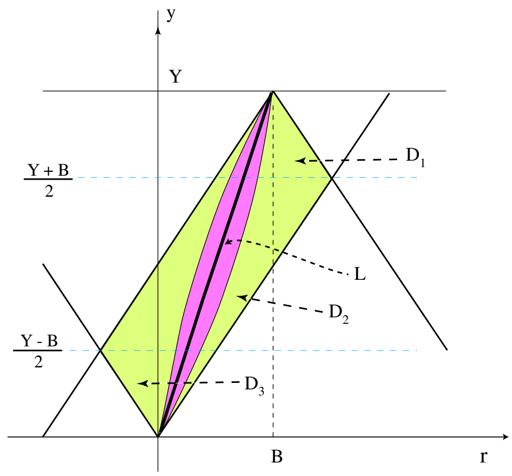

Figure 6: Two intersecting F-cones. Region $\mathbf{D} = \mathbf{D_1} + \mathbf{D_2} + \mathbf{D_3}$ is where the two $\mathcal{F}d$ interact. In the neighbourhood of the line L the maximum number of partons (mostly hard) interact.

of partons from the two $\mathcal{F}d$ collide - and thus from this tube around $\mathbf{L}$ the maximum of the secondary particles (gluon jets) is emitted.

Thus to understand if we have boost-invariance of the transparency T for such mean hard $\mathcal{F}$ configurations we need to consider the behavior of $\Gamma(u_1, u_2, B)$ along the L line with the changing y. From (4.5),(4.7) we see that in the interval of boosts $-B/2 < y < B/2$, that is when disks not fully intercept (the section of $\mathbf{L}$ in the region D_2 on Fig.6), the fastest factor σff doesn't change with y. But the factor S_{ab} variates. On the sections of $\mathbf{L}$ in D_1 and D_3 regions we see the different behavior of $\Gamma \sim \exp\left(\sqrt{y} + \sqrt{Y - B + y}\right)$ that is the growth when we go closer to the edges of $\mathcal{FC}$ along $\mathbf{L}$ and becoming maximal on the ends. And it is opposite to the soft $\mathcal{F}$ picture of previous sections - there we expected that the transparency is maximal at small y - that is in the lab.system. So there is a little hope that the contributions to T, coming from configurations close to the mean one are frame independent. This is clearly seen for the limiting case $B = 0$ at $Y \to \infty$, $y \to \infty$, when the integrals in (4.2) become more simple. Here the contribution to T can be estimated as

$$T(Y, y) \sim e^{-\sqrt{\gamma_1 \ y}} + e^{-\sqrt{\gamma_1 \ (Y-y)}} , \tag{4.9}$$

where $\gamma_1 = 8\delta_1$. This expression is simple to understand, because in the exponent in Eq.(4.9) in fact enter the multiplisities of the hardest saturated subdisks. It is unclear, if it is a real symptom of conflict with the t-unitarity or it is enough that only the dominant contribution in T be frame-invariant.

Here, like in the soft eiconal case, the maximum contribution comes from the rare symmetrical configurations of the $|\varphi \cdot \varphi >$ type, and we end with the frame invariant answer, when $T \sim \exp(-\gamma Y)$.

5. The hard Froissaron in the QCD inspired Regge models.

It would be of some interest to reproduce such a hard partonic structure of $\mathcal{F}$ in terms of a standard regge approach. This can be also useful for an unified description of high energy hadronic data in soft and DIS interactions at the maximal available energies. It is often believed that to achieve this one can simply put in RFT the BFKL pomeron with all higher α_c corrections and all necessary vertices, and supply it with some clever "soft" boundary conditions to include effectively the phenomenological $\mathcal{P}$. This is probably correct and must directly lead to the desired results because the equation (3.1), from that we started, reflects directly the BFKL dynamics. But this way is too general and needs further refinements.

The good starting point can be made from the generalization of the BFKL equation by the inclusion of the running coupling constant, discussed in various papers. The main outcome [23] we need is that in this case $\mathcal{P}$ is represented by the infinite sequence of regge poles $\mathcal{P}_n$ with intercepts $\Delta_n \sim 1/n$, and that u_n - the mean virtualities of $\mathcal{P}_n$ - grow [19] like $\sim n$. The $\mathcal{P}_n$ components with high n are mainly perturbative,

and the components with small n can have a large nonperturbative admixture. This especially concerns to the first $\mathcal{P}_1$ component with a minimal virtuality that can be almost purely nonperturbative, and so can be directly associated with the "low-energy" phenomenological $\mathcal{P}$.

One can hope that such a model, slightly generalizing the usual one -$\mathcal{P}$ regge approach, can be applied for a better phenomenological description of various data, also at existing energies; but it evidently needs additional investigations [9].

The main quantities we need are following. The contribution of components $\mathcal{P}_n$ of such an "unitarised" BFKL-like pomeron can be taken as (1.1) in the simplest factorized form

$$v_n(b,y) \;=\; \frac{g_n \cdot g_n}{\alpha'_n y_1} \cdot \;\exp\Big(\; \Delta_n y_1 - b^2/4\alpha'_n y_1 \;\Big) \quad , \tag{5.1}$$

as corresponding to the generalisation of (1.8) to multi -$\mathcal{P}$ form

$$\mathcal{L} \;=\; \sum_n \;\Big(\; \psi_n^+ \frac{\overleftrightarrow{\partial}}{2\;\partial y}\psi_n \;+\; \Delta\psi_n^+\psi_n \;+\; \alpha'_n\; \vec{\partial_\perp}\psi_n^+ \;\vec{\partial_\perp}\psi_n \;+\; (\psi_n^+ J_n + J_n^+\psi_n) \;\Big) \;+ \tag{5.2}$$

$$\sum_{mnk} r_{mnk}\; (\psi_m^+\psi_n^+\psi_k + \psi_k^+\psi_m\psi_n) \;\;+\;\; ...$$

where :
the n-th pole intercepts are $\Delta_n \sim 1/n$;
the mean virtualities, corresponding to the n-th pole, grow like $u_n \sim n$;
α'_n - slopes of n-th pole $\sim e^{-u_n} \cdot u_n^{-2}$;
r_{mnk} - $3\mathcal{P}$ vertices between the corresponding poles; now not much can be said about them, but one can expect [10] that $r_{mnk} \sim \exp(-u_n)$, where n is the maximum value from (m,n,k),
g_n - are n-th pole vertices that fast decrease with n $(\sim e^{-u})$.

Next we must choose the order of summation of the reggeon diagrams with $\mathcal{P}_n$. If we simply put $\sum v_n$ to the eiconal expression, instead of v, then we become a system of black disks, and it is not a configuration from that we want to start, as it was explained in the previous sections. Perhaps we must firstly take into account the parton recombination, or in terms of $\mathcal{P}_n$ - their joining described by the vertices r_n.

The corresponding minimal mechanism for the reggeon diagrams is presented by the sum of all tree diagrams. In a quasiclassical approximation we can neglect the transverse $\mathcal{P}$ motion inside the tree diagrams. Because we prepare to use the resulting expressions mostly inside the $\mathcal{F}$ disks, where all transverse gradients of the density are small, the inclusion of the real transverse motion can only renormalise the values of parameters, resulting from such an effective zero $\perp$ dimensional reggeon field theory. We also, for a simplification, take into account only transitions between the $\mathcal{P}_n$ with the same n in the process of their joining in a tree. Then the full contribution from the sum of all tree diagrams can be represented in the simple form

$$v_n \rightarrow V_n \;\simeq\; \frac{v_n}{1+\lambda_n\; v_n} \quad , \quad \lambda_n \sim \frac{r_n}{\Delta_n} \;\sim\; e^{-u_n} \;, \tag{5.3}$$

where r_n are proportional to the $3\mathcal{P}_n$ vertices. Then the quantities V_n itself, or their eiconalised combination

$$S_1(b,y) \;=\; \sum_k \; c_k\; \Big(-\sum_n V_n(b,y)\Big)^k \quad , \qquad F_1(b,y) \;=\; i(1-S_1) \;\; , \tag{5.4}$$

[9] One must also slightly "adjust" some properties of $\mathcal{P}_n$, that follow directly from BFKL. The residues of such poles, when calculated from a simple generalisation of BFKL with a running α_s, not aways give positive vertices. One can hope that the correct inclusion of higher α_s terms cure it automatically.

[10] If we try to extract the $3\mathcal{P}$ vertex from gluing 3 BFKL pomerons with (large) virtualities, we don't find the $\exp(-cu_n)$ behavior, but a large value, coming from the infrared region, because the corresponding loop integrals are of the type $\int du \exp(-u)$. The integrals entering the r_{mnk} vertices are $\sim \int du \exp(-u)\chi_m(u)\chi_n(u)\chi_k(u)$, where $\chi_n(u)$ are the internal wave functions of $\mathcal{P}_n$. The mean u coming from such $\chi_n(u)$ are large $\sim n$, but because $\chi_n(u)$ are very flat the factor $\exp(-u)$ is much more essential, and the dominant contribution is again from small u and gives $r_{mnk} \sim 1/u_m u_n u_k$. The above expression for r_{mnk} is in fact a hypothesis - that when "all" higher orders in α_c are summed and also a mixing of the 2-gluon BFKL pomeron with the multigluon bound states is taken into account, then the resulting $\mathcal{P}_n$ states $\chi_n(u)$ will be "sharply" localised in u around the value $\sim n$. I thank A.B.Kaidalov for many comments and discussions of this question.

can be considered as a first approximation to hard $\mathcal{F}$ with a saturation [11].

Next, starting from this point, one can construct higher enhanced and other irreducible diagrams, that transport high vitruality components to the larger B. For the practical phenomenology one possible way is the generalization of the approach presented in [24] by the inclusion of the $\mathcal{P}_n$ components with higher virtualities, that is to use directly V_n, or expressions given by Eq.(5.4) as a first approximation.

Here we will not enter into details of such a construction, but restrict ourselves with one illustration - the structure of inclusive processes in the central region of rapitity corresponding to such a $\mathcal{F}$. For usual soft case the main $2\mathcal{F}$ diagram gives the contribution to the particle density :

$$\rho_1(Y,y) \equiv \frac{d\sigma}{\sigma dy} \sim \frac{y^2(Y-y)^2}{Y^2} \tag{5.5}$$

and the screening diagrams with $\mathcal{F}$ loops are cancelled.
Consider now the inclusive cross-sections at larger virtualities u. It is more convenient and informative to consider them in the representation with definite impact parameter B between colliding particles and transverse distance b at that inclusive particle (object) with rapidity y and virtuality u is measured. The

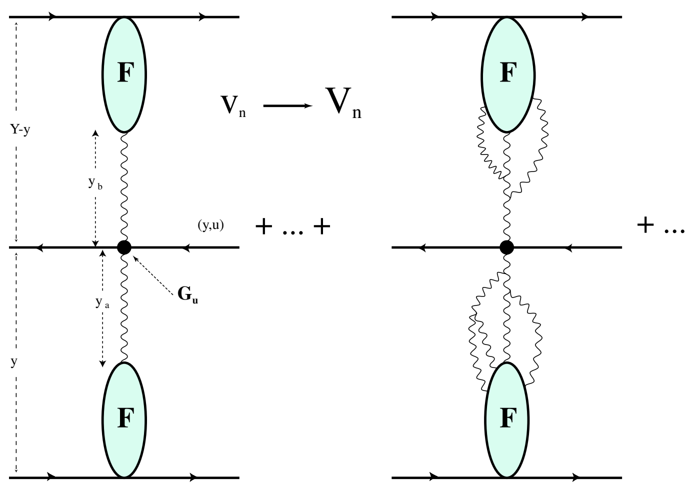

Figure 7: Diagrams for the inclusive production of hard objects with a virtuality u. Wavy lines correspond to $\mathcal{P}_n$ components with $n \sim u$. **F** -blocks correspond to the soft θ -like disks.

main contribution comes from the diagrams Fig.7 :

$$\rho_1(Y,y,B,b,u) = F_2(B-b,Y-y;u)\cdot G_n(u)\cdot F_2(b,y;u) \ , \tag{5.6}$$

where the inclusive vertex $G_n(u) \sim e^{-u}\tilde{g}$ at $n \sim u$ and the "structure function" F_2 are given by

$$F_2(b,y;u) = \int dy_a d^2b_a \ F_1(b_a, y-y_a) \ V_n(b-b_a, y_a) \tag{5.7}$$

and the similar expression for other F_2. For large virtualities u the most essential are the $\mathcal{P}_n$ poles with $n \sim u$. Also, for a large n, the motion in transverse plane is suppressed, so approximately $v_n(b,y;\ n \sim u) \sim \delta^2(b)\ v(u,y)$, where $v(u,y) \sim \exp(\Delta_n y) \sim \exp(\delta y/u)$. For $F_1(b,y)$ we take the approximation to a soft gray disk as $f_0\ \theta(ay-b)$ where $f_0 \sim (1-T_{soft})$. This expression for $v(u,y)$ is valid for such an u, when the corresponding partons are far from a saturation, that is for $y \ll u^2/\delta$. Taking all this into account we have:

$$F_2(b,y;u) = \int dy_a F_1\ v(u,y_a) \sim \theta(y-b)\int^{y-b} dy_a v(u,y_a) \ \sim \ u\ e^{\delta(y-b)/u}\ \theta(y-b) \tag{5.8}$$

[11] Note that this step is the most uncertain in this program - but something equivalent anyway must be done. From one hand one must stop somehow the growth of $\mathcal{P}$ density - to, at least, use safely reggeon diagrams. And, on the other hand, this growth is very likely stopped on $\mathcal{P}$ densities close to the border of the domain of applicability of reggeon diagrams, or at all outside it. Note also that close to the $\mathcal{P}$ saturation region many $\mathcal{P}$ interactions can be essential, and therefore Eq.(5.3) for V_n is only the qualitative one

From here, after substituting in Eq.(5.6), the simple expression for the inclusive density follows:

$$\rho_1(Y, y, B, b, u) \ \simeq \ \tilde{g}(u)\ e^{-u+\delta(Y-b-|B-b|)/u}\ \theta(y-b)\ \theta(Y-y-|B-b|) \tag{5.9}$$

It is remarkable that this ρ_1 in fact doesn't depend on b - it is because hard particles are produced from the edge of the soft disk F_1.

When $y > \eta(u) \simeq u^2/\delta$, than the saturation at the u scale begins, and we approximate it by $v(u, y) \rightarrow V = v/(1 + \lambda_u v)$ in F_2. As a result F_2 stops to grow at $v_{max} \sim \lambda_u^{-1} \sim e^u$.
Using this substitution we can write the approximate expression, valid in all the regions of virtualities:

$$\rho_1(Y, y, B, b, u) \ = \ \frac{G_u(u)\ v(u, y-b)\ v(u, Y-y-|B-b|)\ \theta(y-b)\ \theta(Y-y-|B-b|)}{(1+\lambda(u)v(u, y-b))\ (1+\lambda(u)v(u, Y-y-|B-b|))} \simeq$$

$$\simeq \frac{\tilde{g}(u)\ e^{-u+\delta(Y-b-|B-b|)/u}}{(1+e^{-u+\delta(y-b)/u})\ (1+e^{-u+\delta(Y-y-|B-b|)/u})}\ \theta(y-b)\ \theta(Y-y-|B-b|)\ , \tag{5.10}$$

that represents the mean structure of final states produced in collisions of the enclosed multi-$\mathcal{F}$ disks, considered in the previous sections.

It is also not so complicated to write the expression for ρ_1 , in that the soft disk is not θ-like, but more smooth - as "normal" amplitude profiles resulting from several $\mathcal{P}$ exchanges at not "too asymptotic" energies.

6. Brief discussion of some connected questions

We see that QCD leads to a natural picture of $\mathcal{F}d$, that asymptotically becomes more and more black, due to the growth of the hard parton component with energy. And it is probably the way how the main requirements from the t and s-unitarity can be fulfilled.
In this Section we briefly mention some other interesting aspects of the $\mathcal{F}$ behavior, that, so or another way, are connected to the properties of $\mathcal{F}d$, considered in the previous sections.

- The most often raised question in the connection to $\mathcal{F}$ is if, at existing energies, we are far away from the region in that the $\mathcal{F}$ type behavior starts, or may be one can already find some evident signs of it ? We don't know this certainly, because there is no common point of view, what are parameters entering the calculation of amplitudes. If the mean number of the $\mathcal{P}$ exchanges $n(s)$ at $\sqrt{s} \sim 1TeV$ is $\simeq 1$ and the multi -$\mathcal{P}$ exchanges are somehow suppressed, as for example advocated in [25], then we must evidently go to much higher energies even to come near to the $\mathcal{F}$ behavior. But in most other approaches - like the dual topological string model (see review [26]), that gives a good description of a very broad set of data, one usually has $n(\sqrt{s} \sim 540GeV) \sim 2-5$, and the calculated transparency is ≤ 0.1 at $B \leq 1 \div 2GeV^{-1}$. In such a case we already have in the middle of the fast hadron a clearly seen embryo of the $\mathcal{F}$ disk.

  But even if we are optimistic about a possible fast appearance of $\mathcal{F}$, the continuation of such treatments to much higher energies does not give such large radii of the $\mathcal{F}$ disks, that the regularities described in the previous section can be clearly seen. For example, at Planck energies (that are in some sense critical, because at higher energies new mechanisms can appear) one can estimate that the $\mathcal{F}d$ radius $R_F(\sqrt{s} \sim 10^{19}GeV) \sim (20-30)GeV^{-1} \sim 5-6fm$, depending on parameters [12]. Even here the $\mathcal{F}$ radius is close to that of heavy nuclei. Also the mean transverse momenta of partons in $\mathcal{F}d$ should be expected rather small - much smaller than in DIS at already existing energies.
  One can also expect, that the events with the final multiplicity much higher than the mean one are in many respects close in structure to that one coming from an asymptotic $\mathcal{F}d * \mathcal{F}d$ collision. The estimates of properties of such events from multi-$\mathcal{P}$ exchange models show also in this direction. But, at the same time, we know that the data for such high multiplicity events don't show the growth of $< k_\perp >$. So, if the $\mathcal{F}d$ component already starts to grow, it is so far only a soft disk.

- The heavy nuclei interactions at high (but acceptable on the accelerators) energies can also have many common points with the collision of $\mathcal{F}$ disks. If we consider the interaction at sufficiently high s.c.m.

[12] The limiting value of R_F alloved by the axiomatic limitation is only $\sim 3-5$ times larger, because it is defined by the π-meson scale. But the "real" growth of the R_F in QCD (and in regge phenomenology) is defined by the larger scale $\sim 0.5 \div 1GeV$

energy (say 200 GeV per nucleon, like expected in LHC), then the longitudinal sizes of heavy colliding nuclei become $\sim 1/20 \ll GeV^{-1}$. In this configuration the low energy parts of parton clouds from the individual nucleons fully overlap in the longitudinal directions, and as a result the density of low momenta partons can essentially exceed the saturation limit for soft partons. Then number of things can happen. Or all "superfluous" partons will be simply absorbed and their mean density approaches to the saturation limit for the soft virtuality scale. Or the "additional" partons will be pushed to the region of higher virtualities, remembering the hard part of $\mathcal{F}d$ at more higher energies.
It should be noted however, that for such a longitudinal joining of parton clouds we don't need too high energies - it can take place at energies, already reached in A*A collisions. But no growth of the mean $k_\perp$ is seen. Probably it means, that even using this $A^{1/3}$ multiplication of a wee parton density we do not yet reach the saturation scale even for soft partons.

- At the end let us discuss the question about a possible limitation from *above* on the energies, where $\mathcal{F}$ behavior can be applied. And what can be expected for the after-$\mathcal{F}$ behavior, if the local field theory is only an approximate one?
When we initially are inside the region of applicability of the local field theory (QCD) and the possible energies are "in our hands", then some changes in the $\mathcal{F}$ behavior can be expected only when the virtualities essential in $\mathcal{F}$ amplitudes grow with energy and finally reach the limiting flied theory scale (some physical cutoff connected with the Plank mass, the string scale,...). It is possible that in this case any visible changes in $\mathcal{F}$ behavior will not appear at all, if dominant processes remain somehow soft at all energies, or the influence of such small scale processes can be not too essential. For example, in terms of the hard disk parton picture (of Section 3), it can change some properties of the hard part of the $\mathcal{F}d$ with the virtuality $u \sim \log G^{-1}$, and, as a result, the cross-sections for the particles production at corresponding $p_\perp$. But it will not influence the $\mathcal{F}$ behavior of the total inelastic cross-section and many other cross-sections of soft processes, because they are controlled by the soft $\mathcal{F}d$ partons.
The other possibility is when some interaction (like the gravitational one), now very week, can grow with energy and become dominant, so as to interfere also with the soft part of QCD interaction. One can expect that changes in the character of the inelastic scattering can appear near the Plank scale $\sqrt{s} \sim 10^{19} GeV$, where the gravitational interaction starts to be equally important. The gravitational cross-sections grow like $\sim G^2 s$ in the perturbative region (see for example [27]). But, as it was proposed in [28], the same character of the growth of the σ_{in} can remain also at the super-planck energies, where $Gs \gg 1$, due to the nonperturbative creation of a large (with a radius $\sim G\sqrt{s}$) events horizon, in a collision process, that absorbs the colliding particles and at the end transforms in a black hole. The similar asymptotic behavior of cross-sections is predicted [16] for strings. But it is essential that in these cases, as opposite to QED and QCD, we can't prepare an analogue of neutral states to separate somehow the long range elastic and bremstrahlung part of scattering.
Evidently such questions need an additional investigation to become more or less clear - but it is not excluded, that after some clever separation of the elastic component, and the adequate quantum-mechanical treatment of the event horizon creation, and also after imposing t - unitarity we end again with the same $\mathcal{F}$ -type of the answer for cross-sections.
It is interesting to note that in the light of recent propositions [29] that the real Planck and string scales can be close to the TeV region, and not to the $10^{19} GeV$ - this question can in principle become available also to experimental investigations. In such a case the gravitational contribution to the inelastic cross-sections can coincide with the hadrons strong interactions already at $\sqrt{s} \sim 10^3 \div 10^4 GeV$ - it is close to the lab.energies $10^{21} \div 10^{23} eV$, at that some strange phenomena in cosmic rays are discovered.

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